# Faster GPS via the Sparse Fourier Transform

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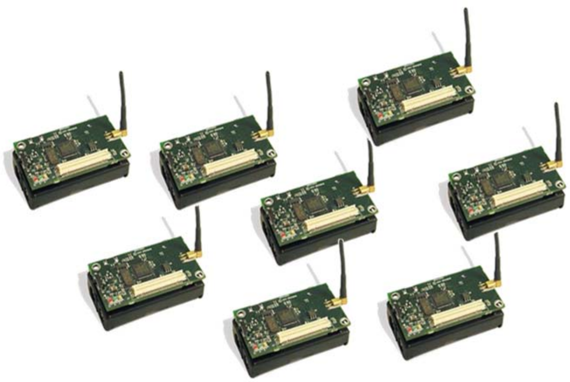
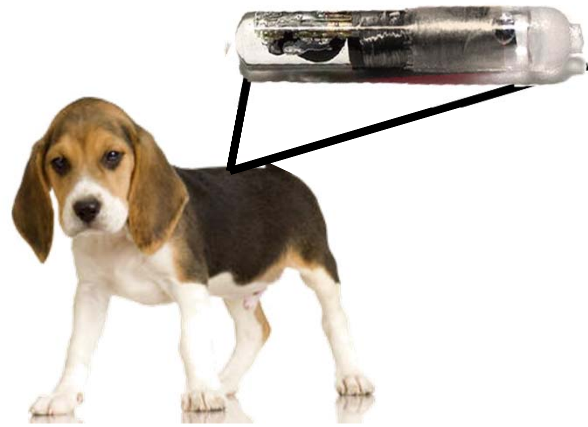
# GPS Is Widely Used







Faster GPS benefits many applications

How Do We Improve GPS?

Need to Improve GPS Synchronization

# GPS Synchronization

Synchronization is locking onto a satellite's signal

- Consumes 30%-75% of GPS receiver's power [ORG447X datasheet, Venus 6 datasheet]

GPS signals are very weak, less than -20dB SNR

**100s of millions of multiplications**

[Team, Kaplan]

# Goal

**Faster Synchronization Algorithm**

Reduce number of operations

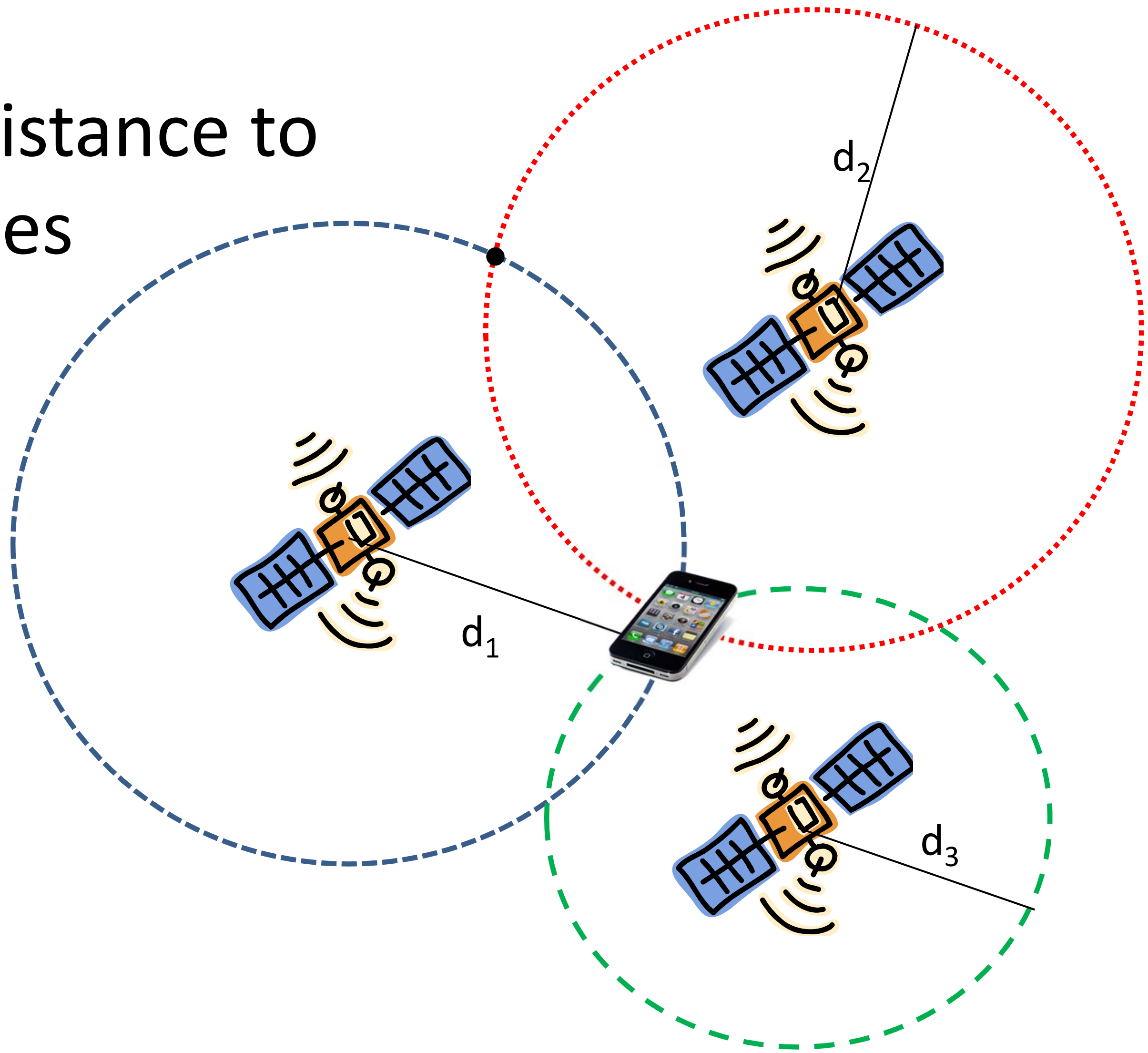
**Reduction in power consumption and delay**

# Rest of this Talk

- GPS Primer
- Our GPS Synchronization Algorithm
- Empirical Results

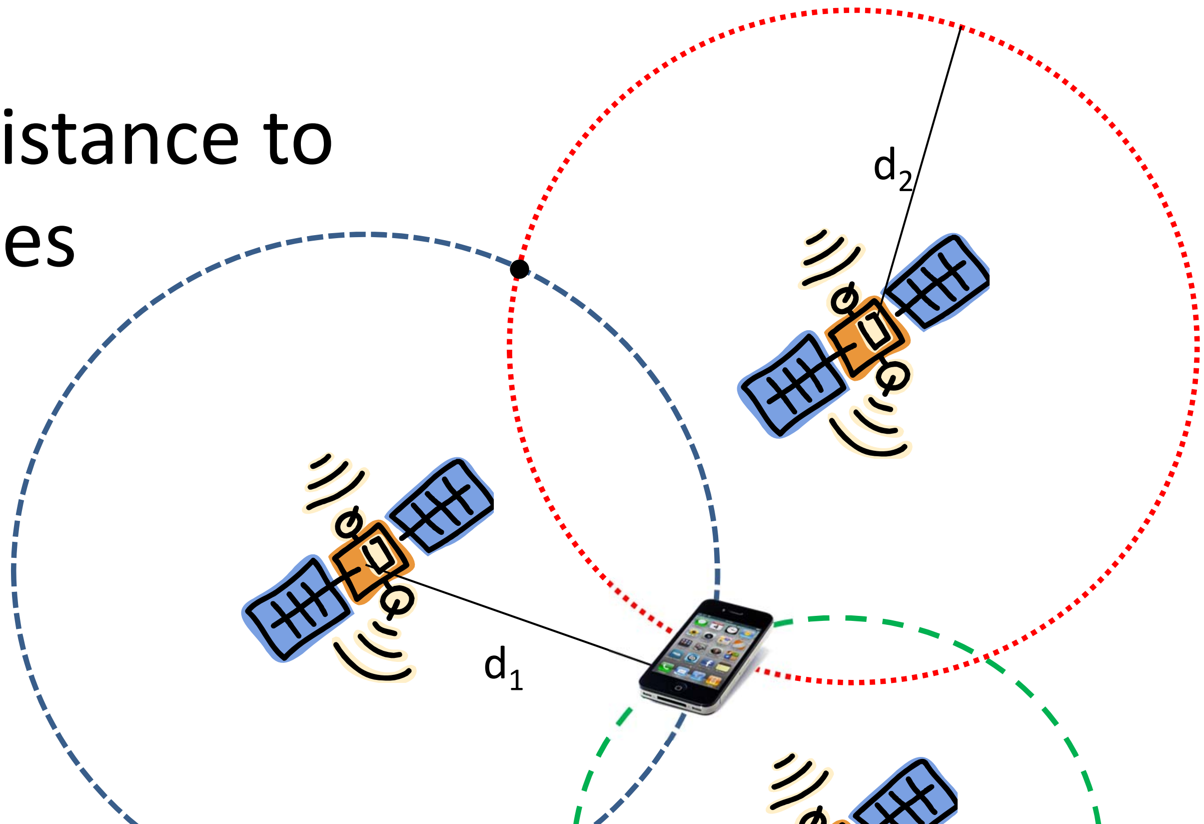
# How Does GPS Work?

Compute the distance to the GPS satellites



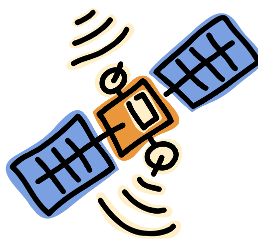
# How Does GPS Work?

Compute the distance to the GPS satellites




**distance = propagation delay × speed of light**

# How to Compute the Propagation Delay?




Satellite Transmits CDMA code

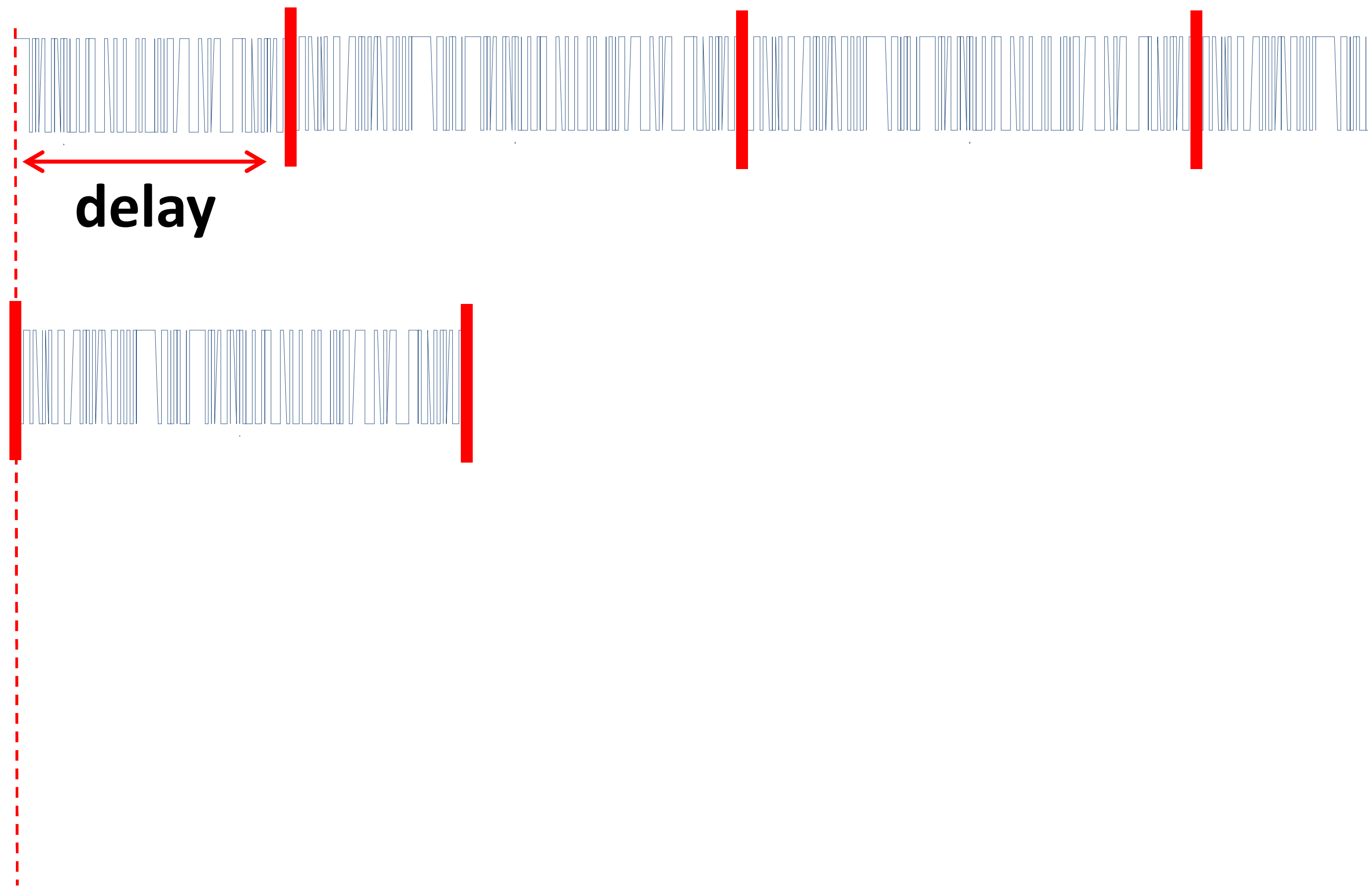
# How to Compute the Propagation Delay?


Code arrives shifted by propagation delay

# How to Compute the Propagation Delay?

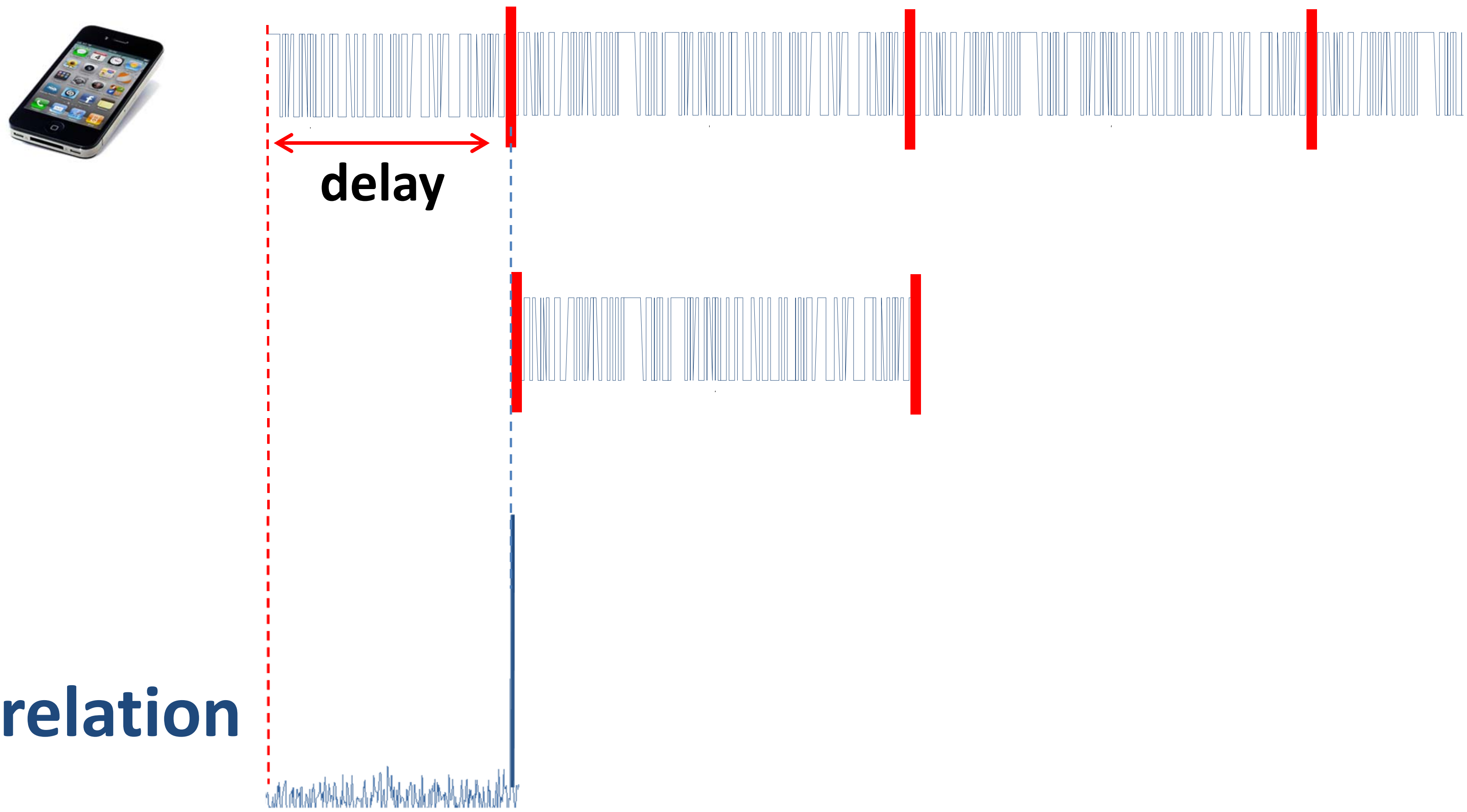

Receiver knows the code and when the satellite starts transmitting

# How to Compute the Propagation Delay?



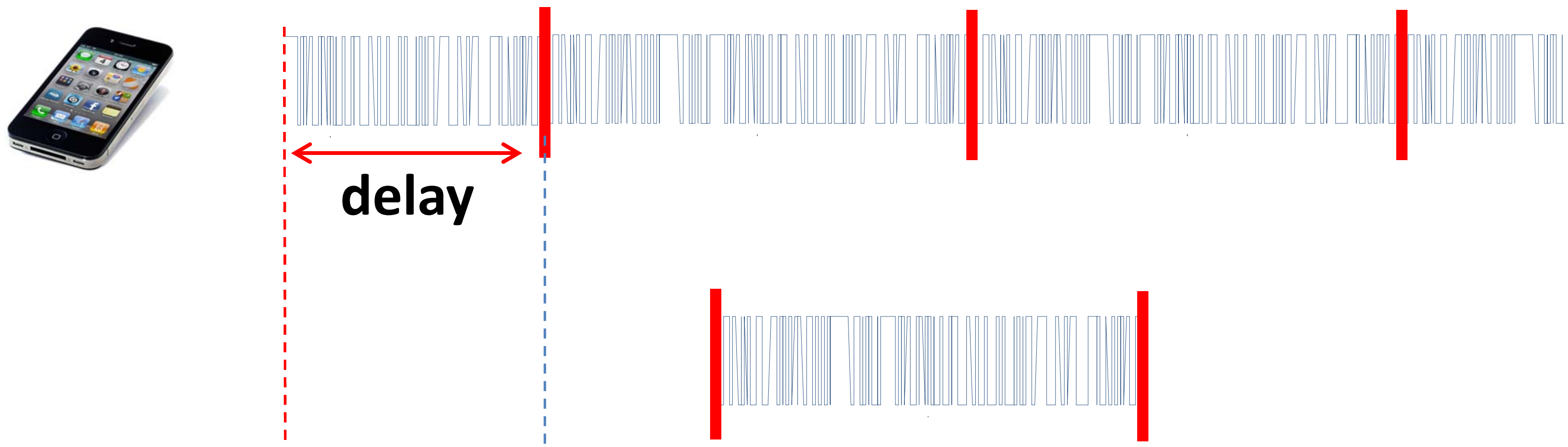

## Correlation

# How to Compute the Propagation Delay?

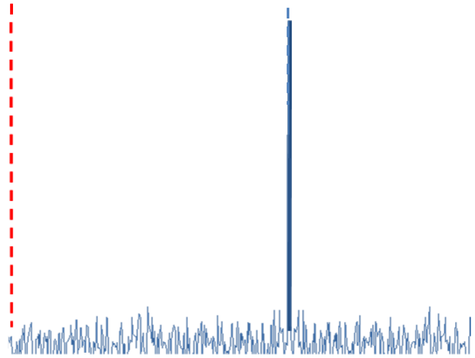



**Correlation**

# How to Compute the Propagation Delay?




Correlation



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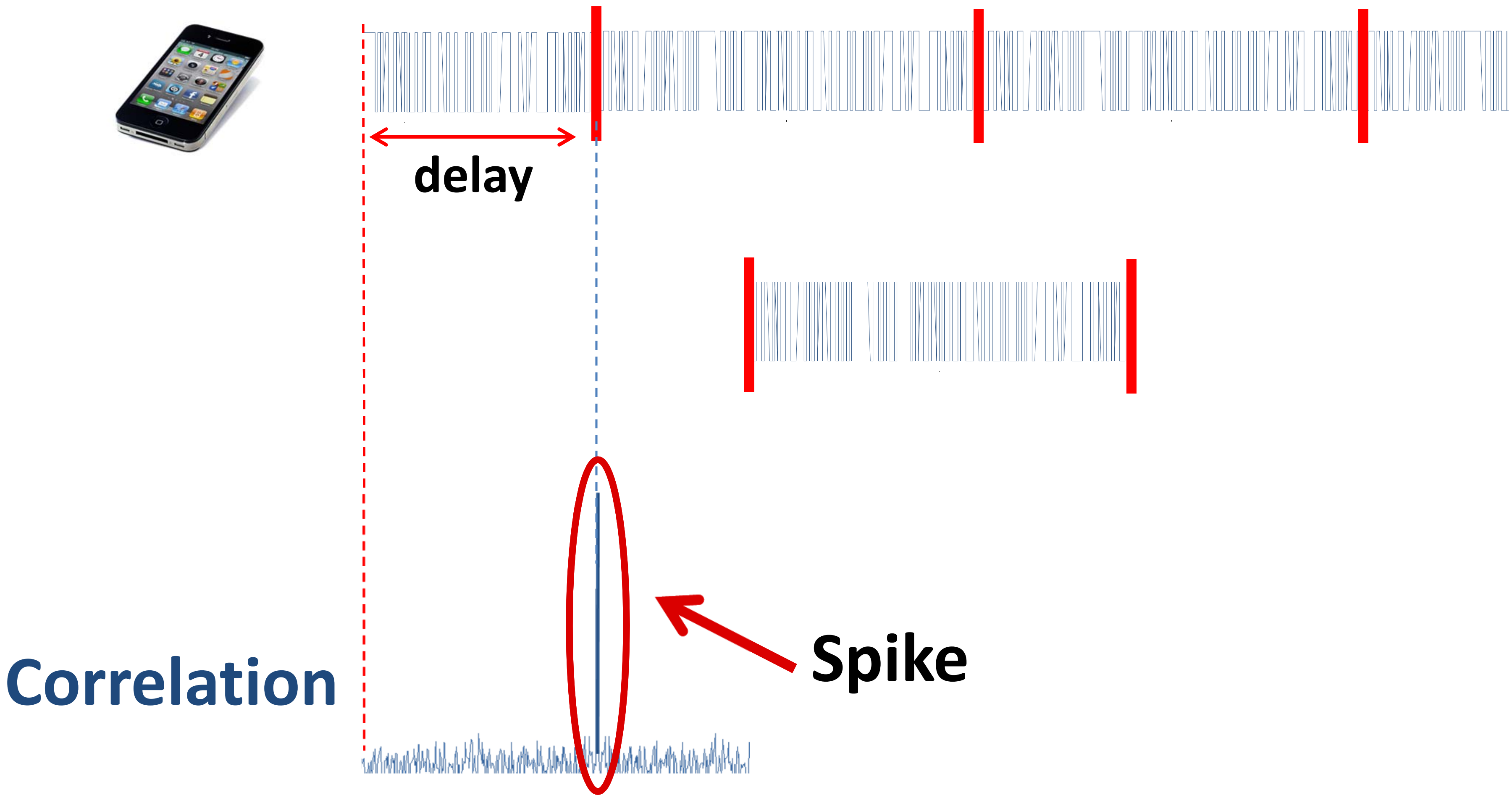


Spike determines the delay

GPS Synchronization is a convolution with CDMA code

**Convolution in Time** ⟺ **Multiplication in Frequency**

$O(n^2)$ $O(n \log n)$

$n$ : Number of samples in the code

GPS Synchronization is a convolution with CDMA code

**Convolution in Time** ⟺ **Multiplication in Frequency**

$O(n^2)$ $\qquad$ $O(n \log n)$

**State of the art GPS synchronization algorithm: $O(n \log n)$**

# Rest of this Talk

- GPS Primer
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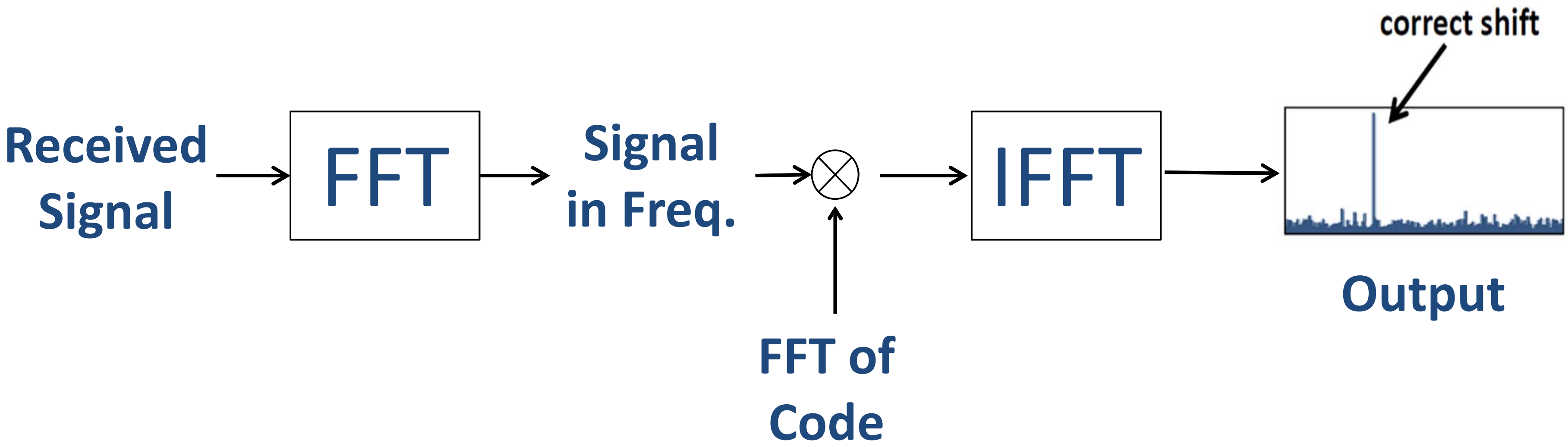
- Empirical Results

# QuickSync

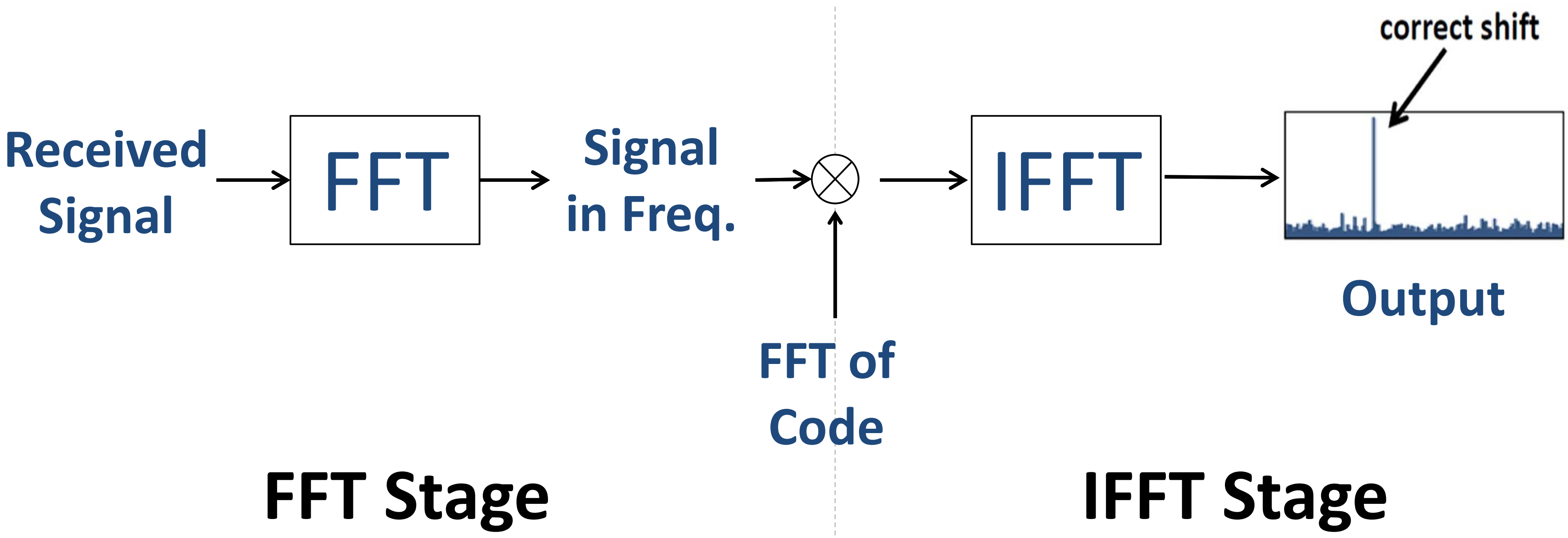
- Fastest GPS synchronization algorithm to date
- Analytical complexity:
  - $O\left(n\sqrt{\log n}\right)$ for any SNR
  - $O(n)$ for moderately low SNR
- Empirical Results:
  - Evaluated on real GPS signals
  - Improves performance by 2.2x

## How can we make GPS synchronization faster than FFT-Based synchronization?

# FFT-Based GPS Synchronization



# FFT-Based GPS Synchronization



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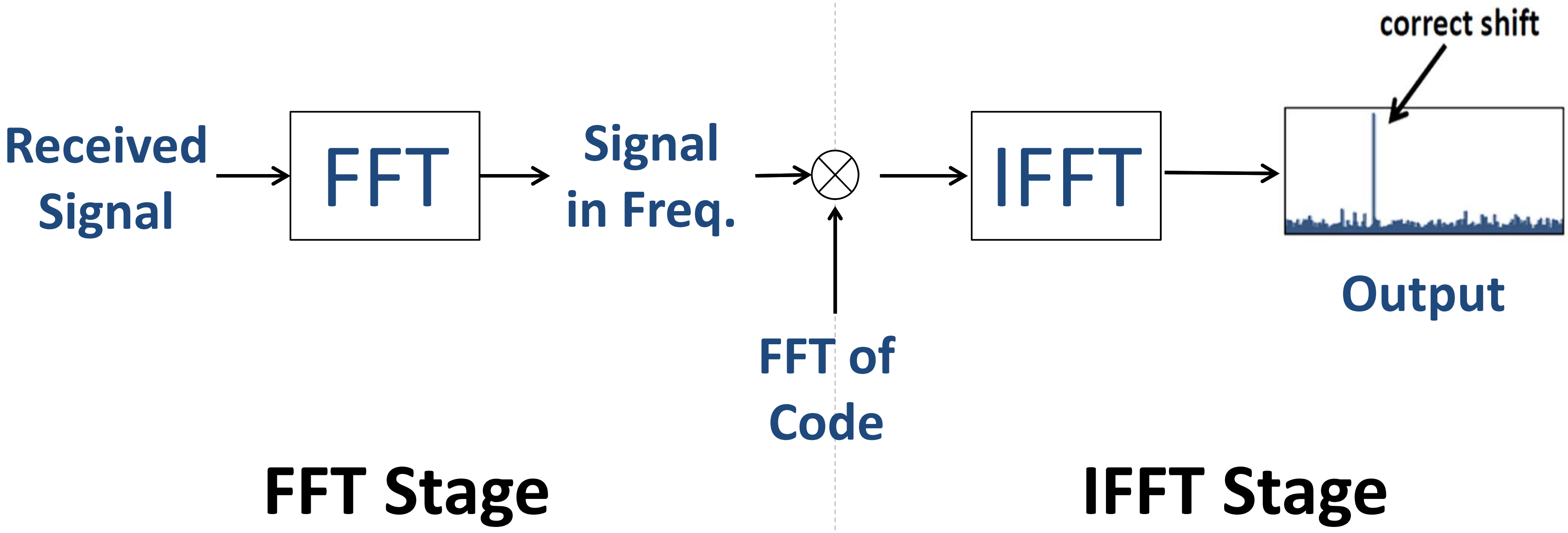
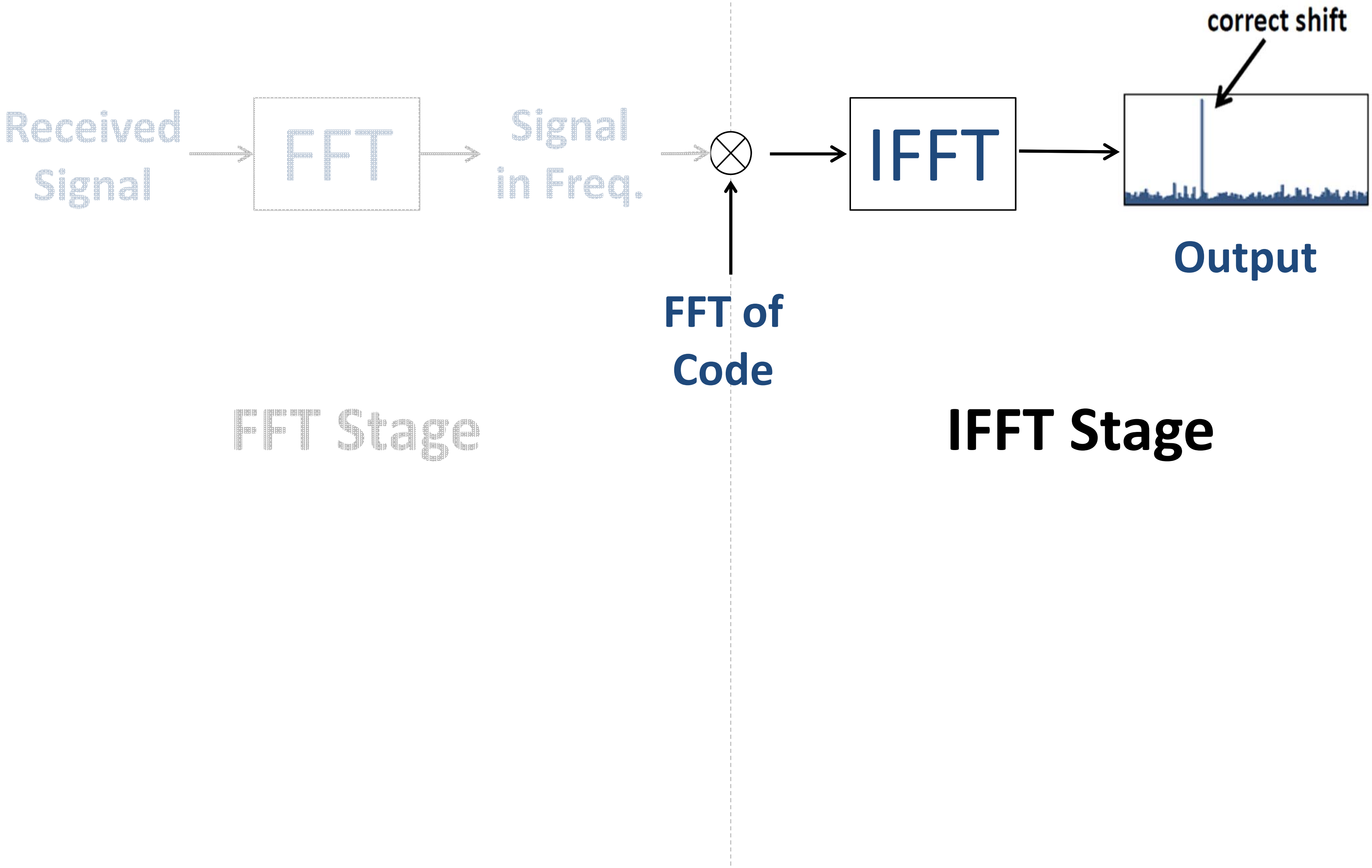



**Each stage takes $O(n \log n)$**

**→ need to reduce complexity of both stages**

# FFT-Based GPS Synchronization



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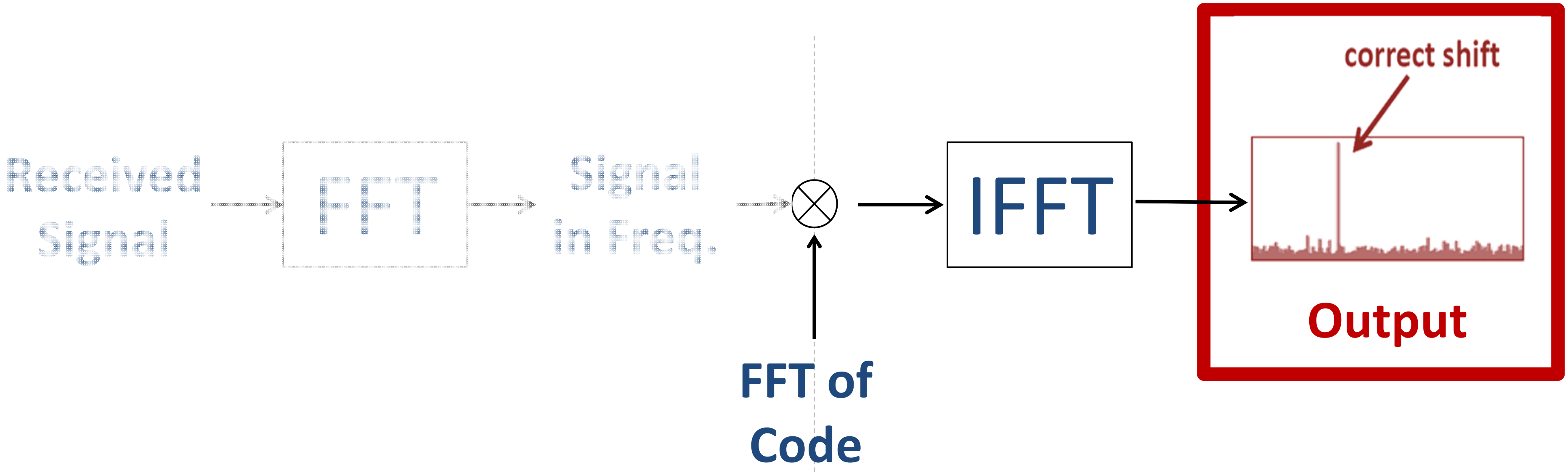
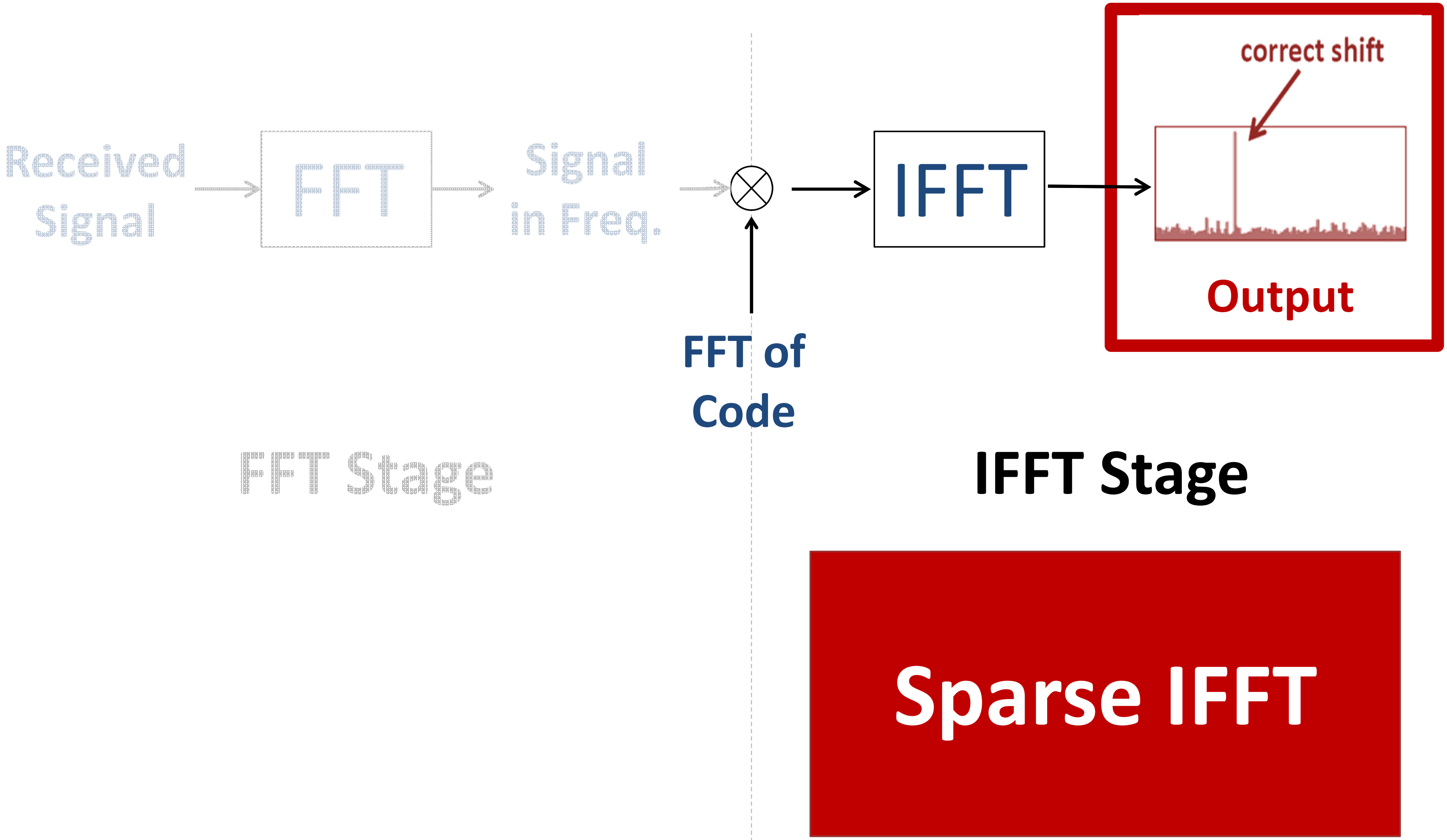



FFT Stage

IFFT Stage

**Sparse**

# FFT-Based GPS Synchronization



# QuickSync

## A Sparse IFFT algorithm customized for GPS

- Exactly One Spike ➔ Simpler algorithm
- Extends to the FFT-stage which is different (will discuss later)

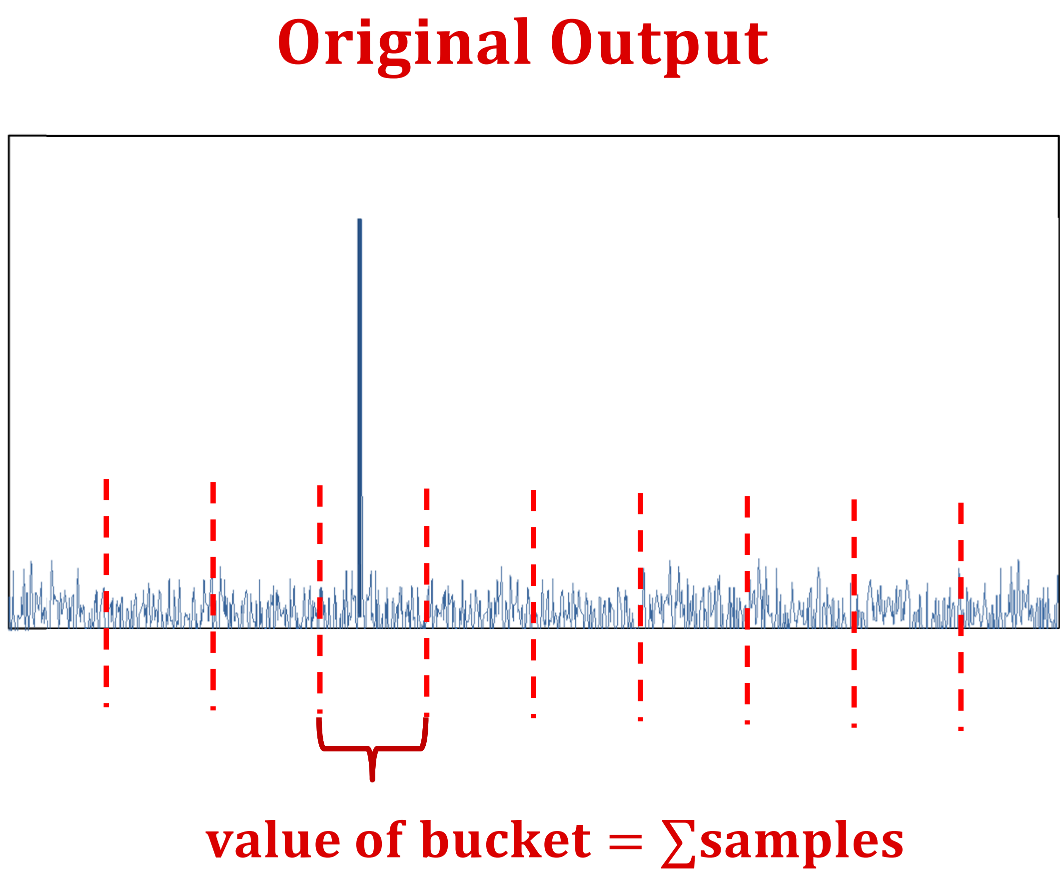
# QuickSync's Sparse IFFT

## 1- Bucketize

Divide output into a few buckets

## 2- Estimate

Estimate the largest coefficient in the largest bucket



# QuickSync's Sparse IFFT

## 1- Bucketize

Divide output into a few buckets

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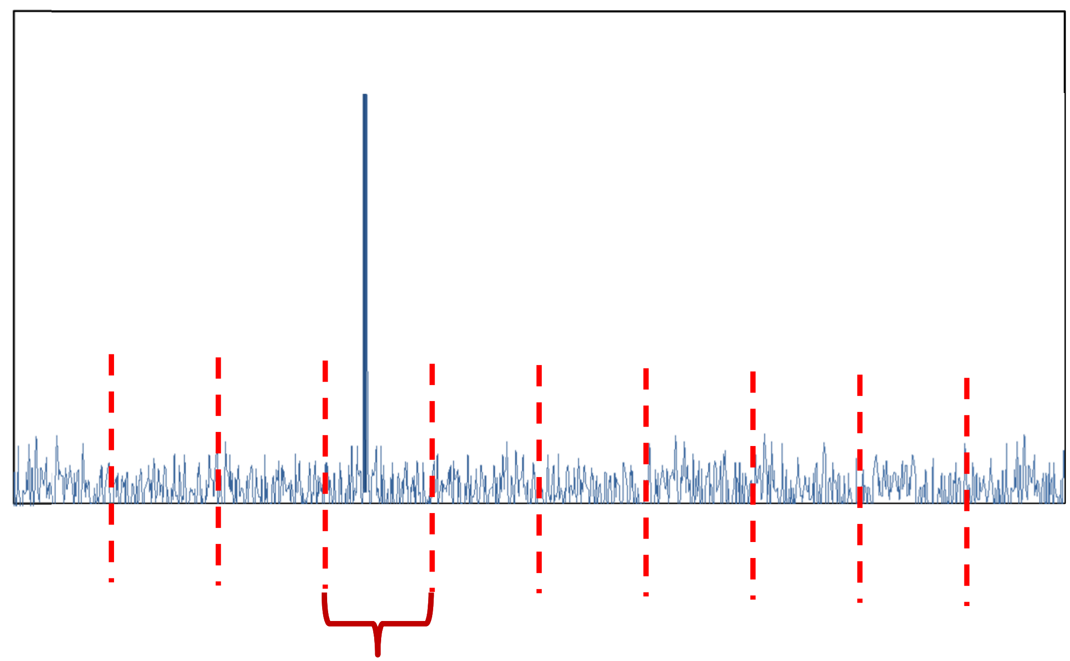
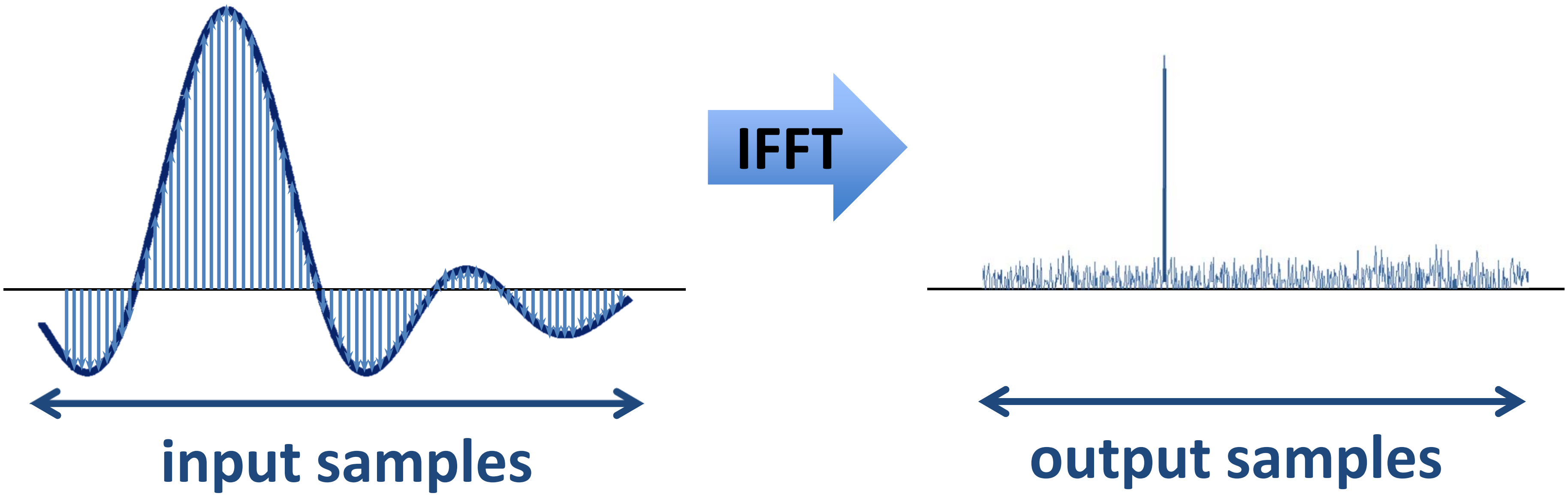
Original Output



value of bucket = $\sum$samples

**So how can we bucketize and estimate efficiently?**

# How to Bucketize Efficiently?



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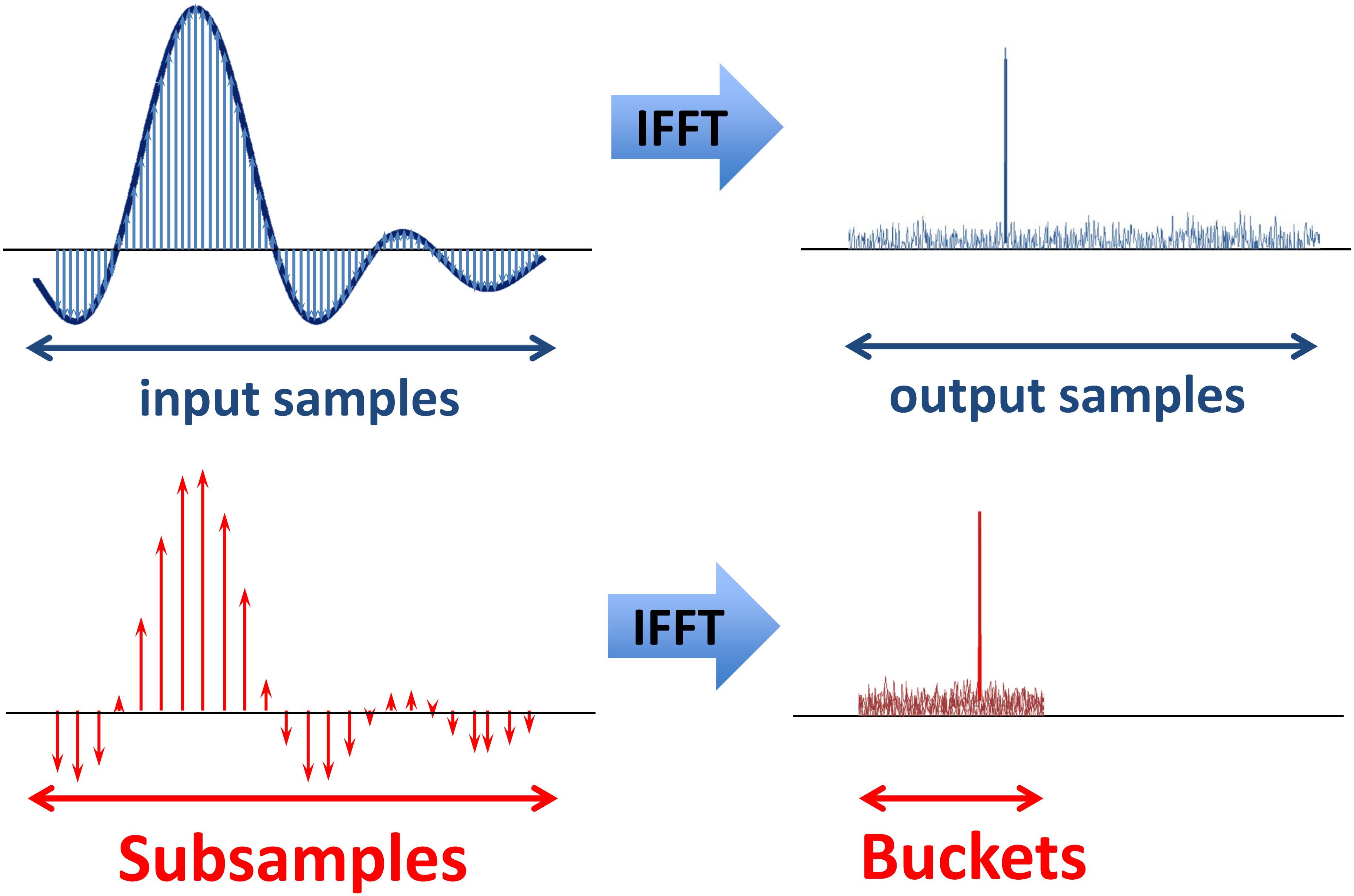

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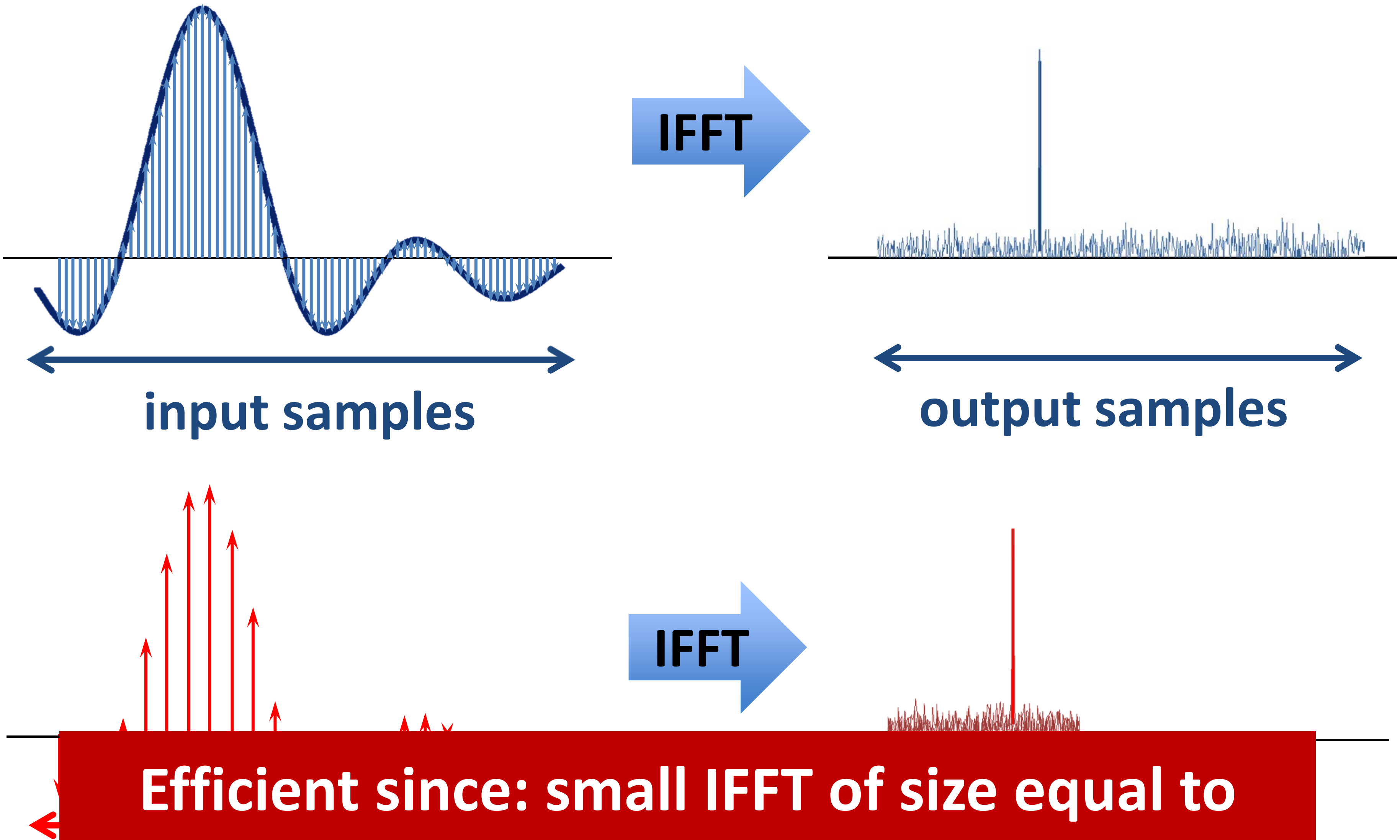
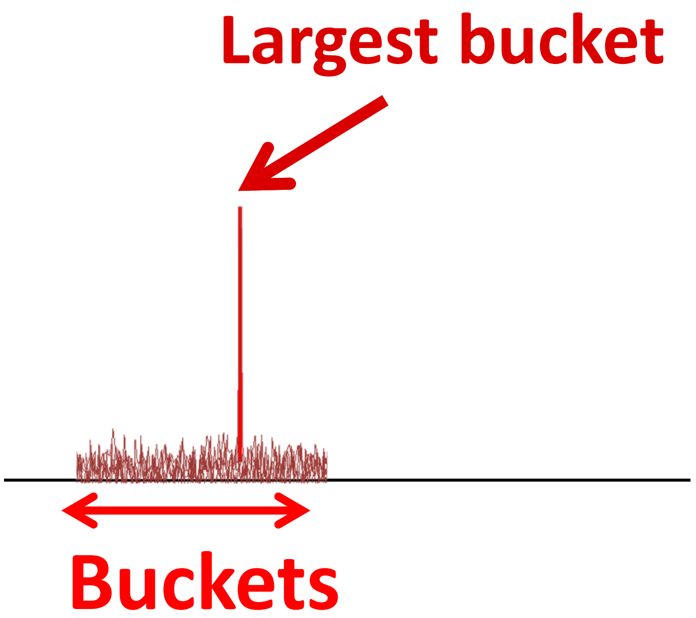



**Efficient since: small IFFT of size equal to the number buckets**

# How to Estimate Efficiently?

- Keep largest bucket; ignore all the rest
- Out of the samples in the large bucket, which one is the spike?



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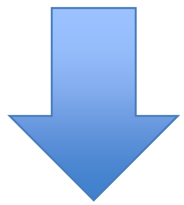
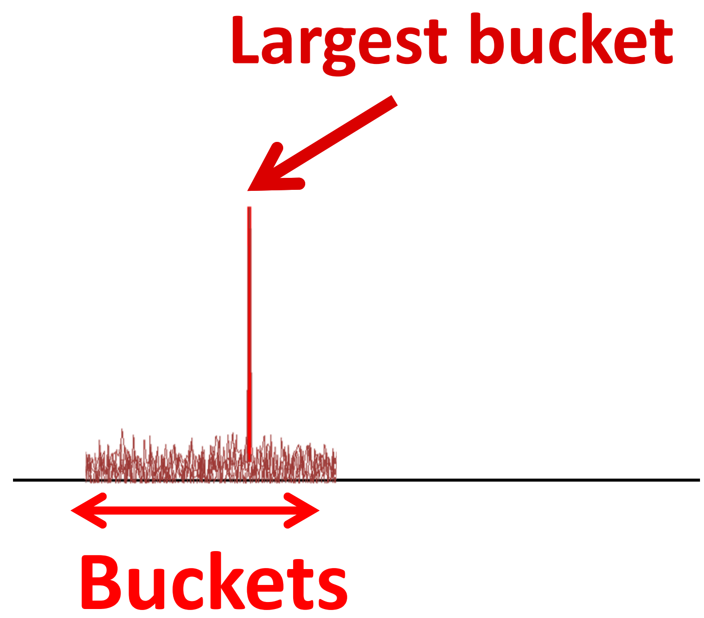


**The spike is the sample that has the maximum correlation**



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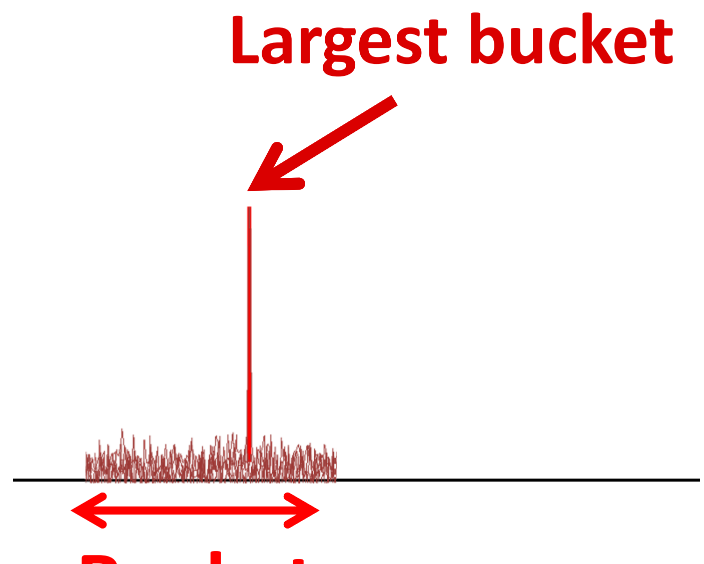



**The spike is the sample that has the maximum correlation**

**Efficient since: compute correlation only for few samples in the largest bucket**

# QuickSync's Sparse IFFT

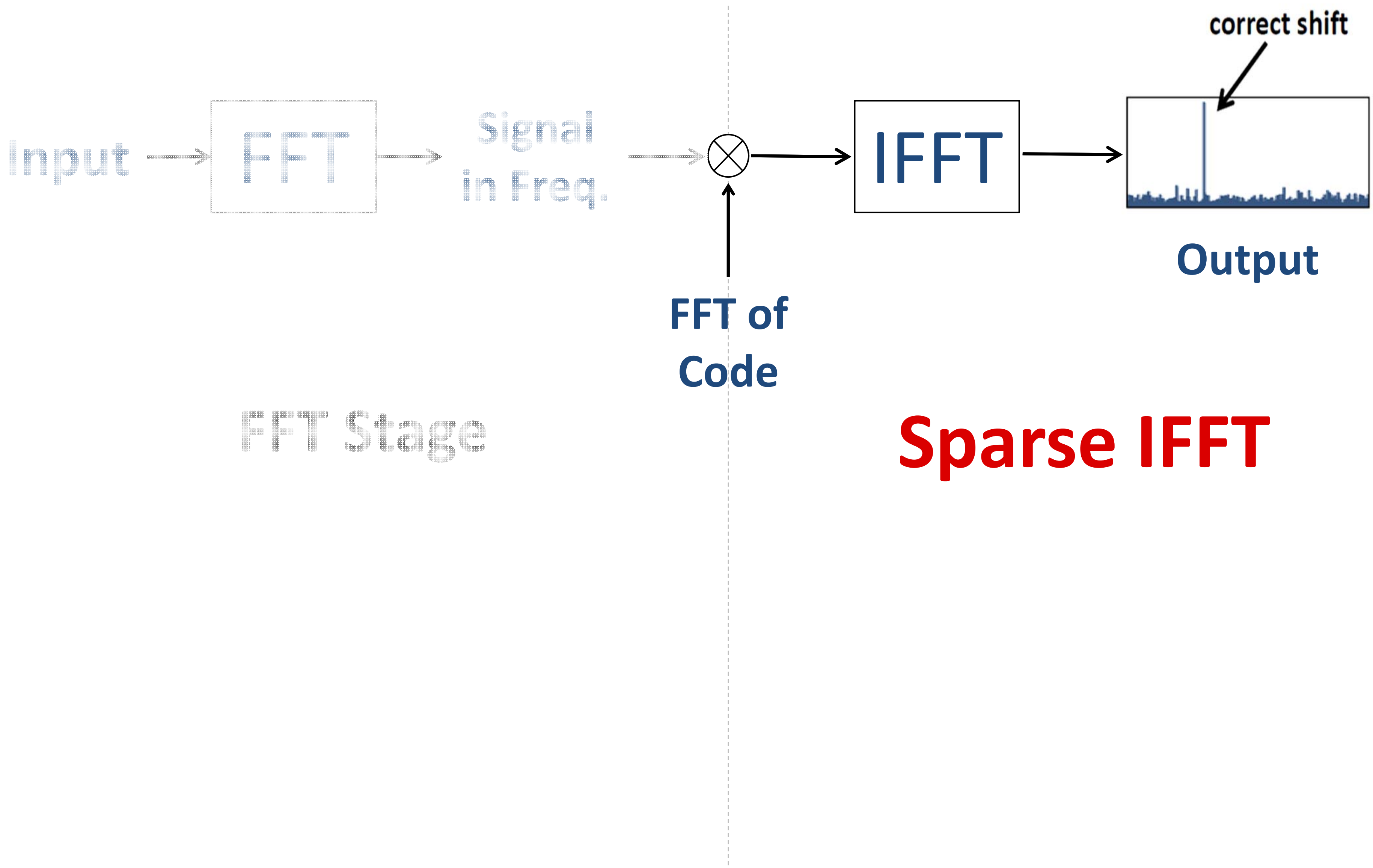
- $n$ is number of samples
- $k$ samples per bucket → $n/k$ buckets

**Bucketization:** $n/k\ \log(n/k)$

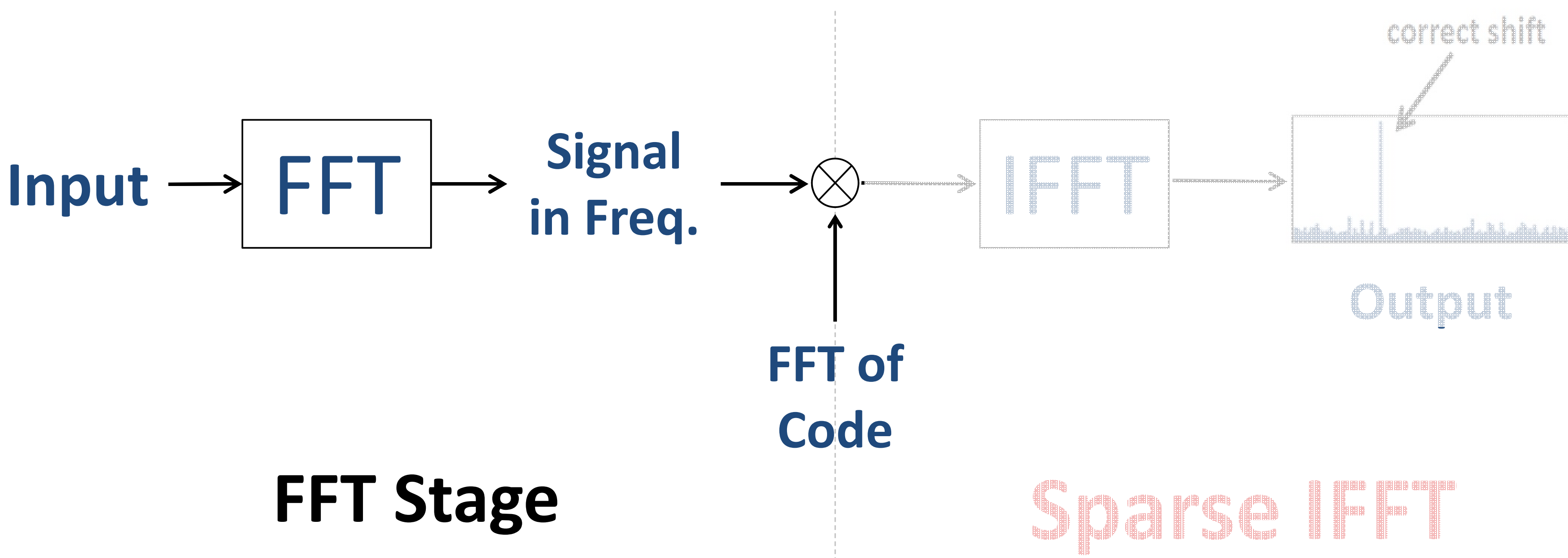
**Estimation:** $k \times n$

$$k = \sqrt{\log n} \quad \Rightarrow \quad O\left(n\sqrt{\log n}\right)$$

# QuickSync Synchronization



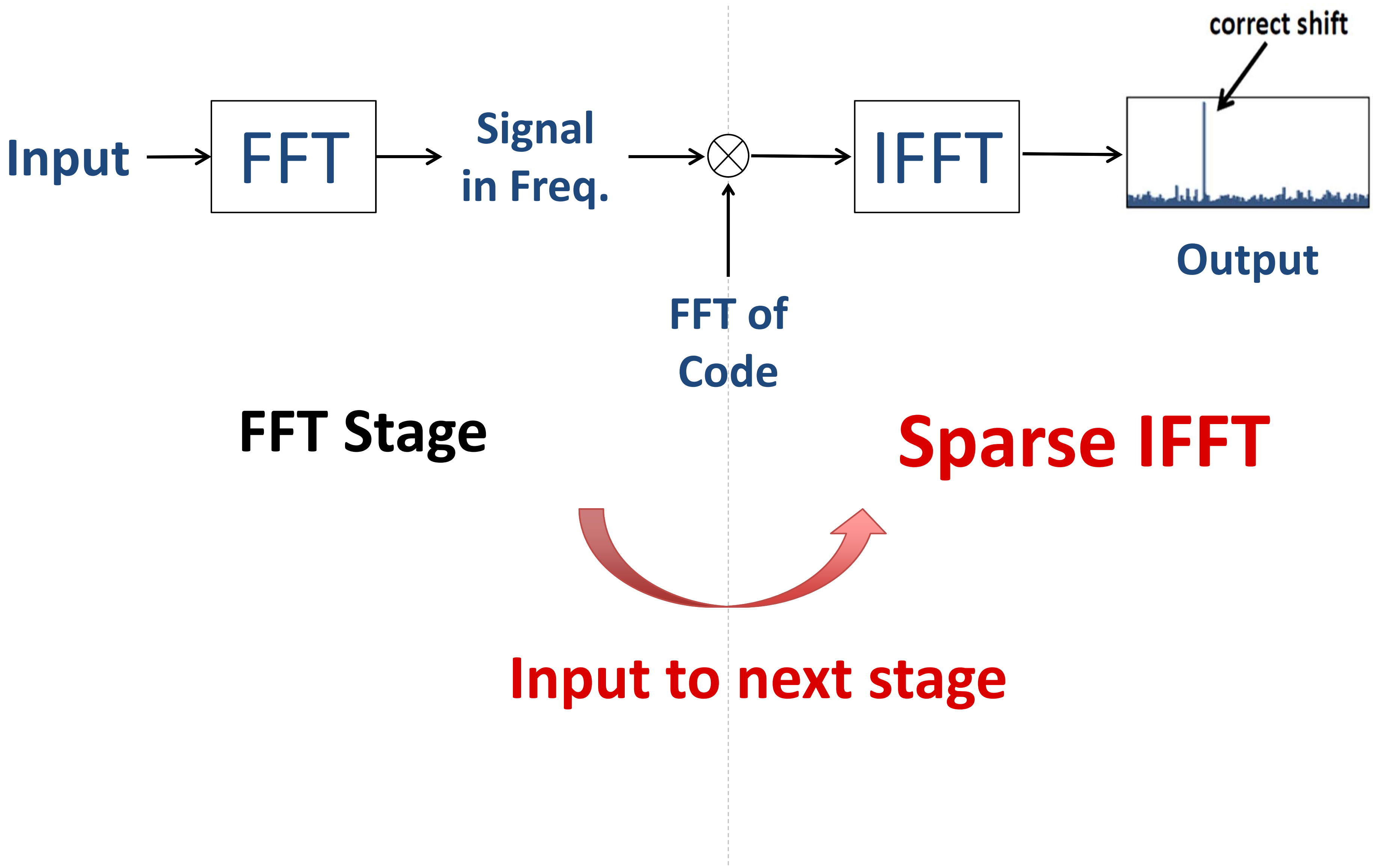
# QuickSync Synchronization




Output is not sparse

Cannot Use Sparse FFT

# QuickSync Synchronization



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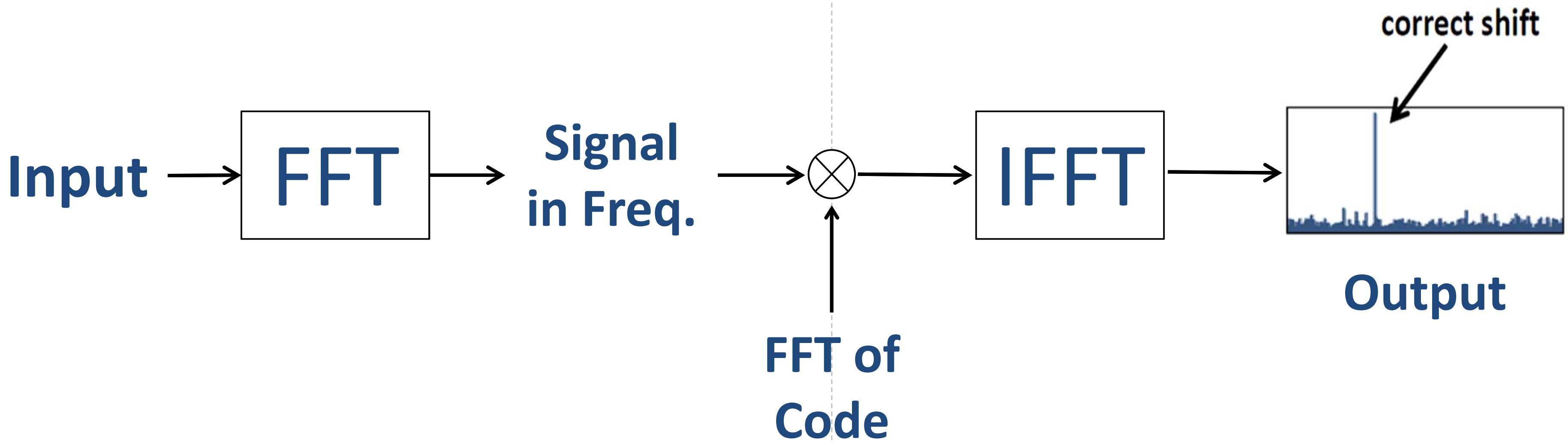


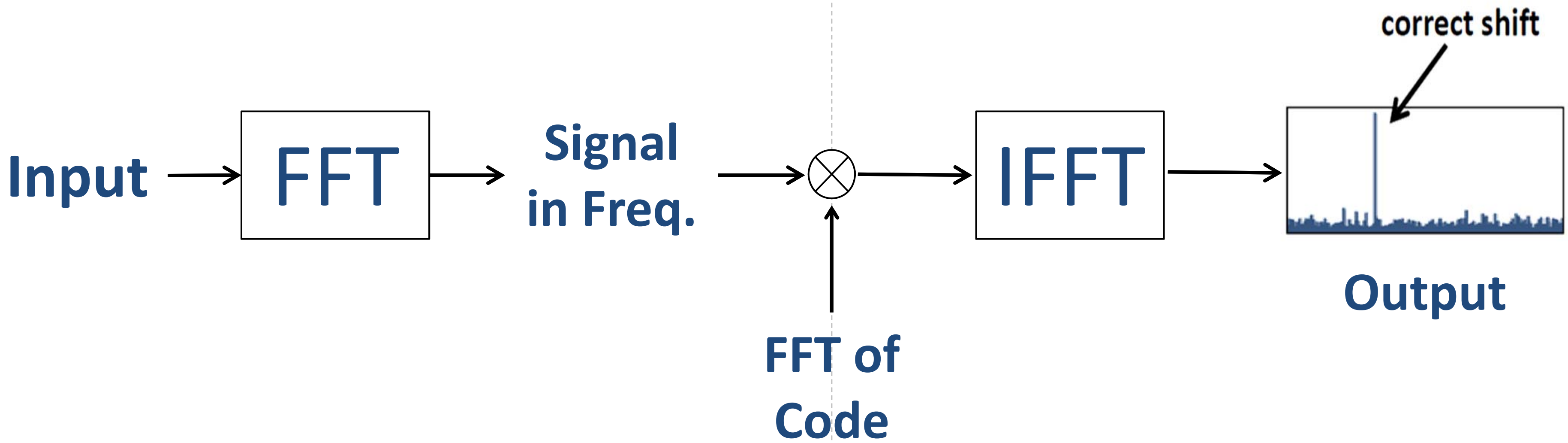

**Subsampled FFT**

**Sparse IFFT**

**Need only few samples of FFT output**

**IFFT samples its input**

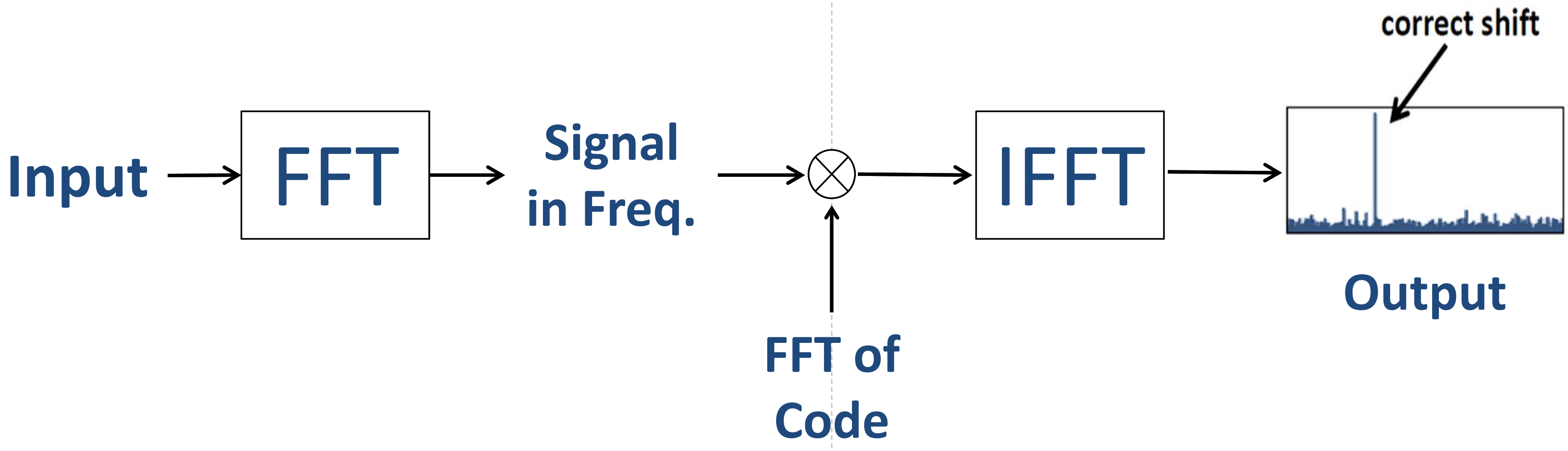
# QuickSync Synchronization




**Subsampled FFT** **Sparse IFFT**

FFT and IFFT are dual of each other

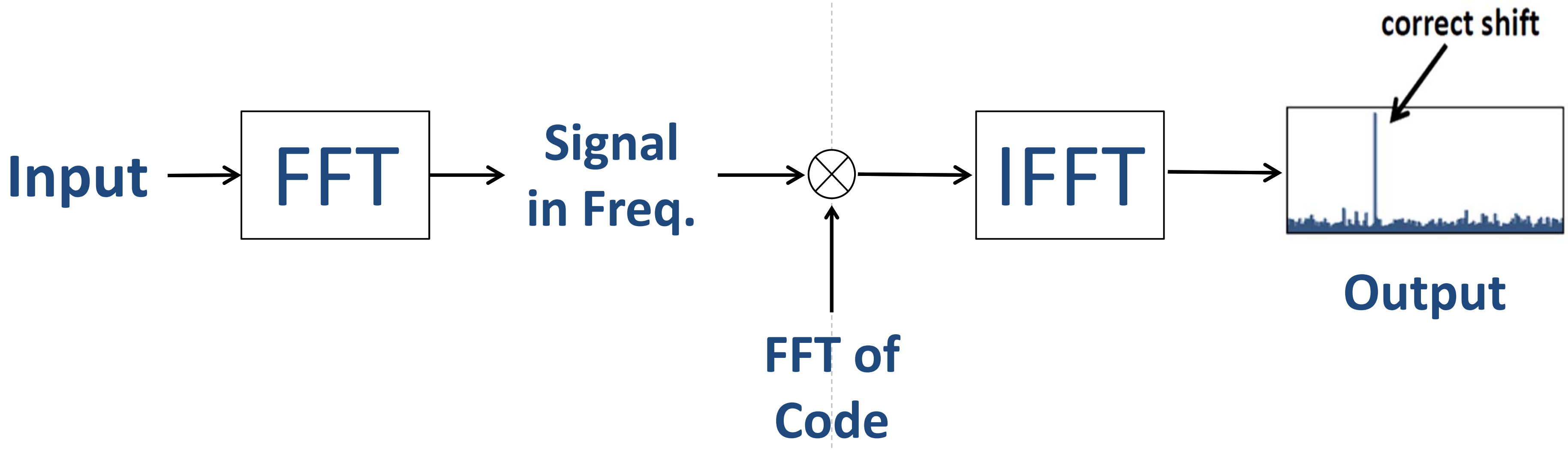
# QuickSync Synchronization




**Subsampled FFT** | **Sparse IFFT**

Bucketization $\xrightarrow{\text{FFT}}$ Subsampling $\xrightarrow{\text{IFFT}}$ Bucketization

# QuickSync Synchronization



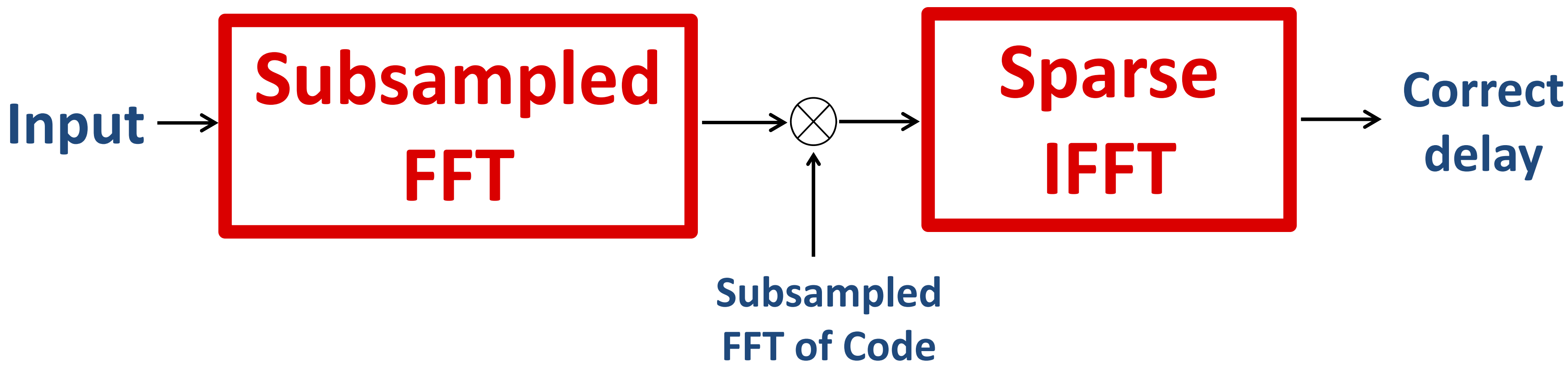

**Subsampled FFT**

$$O\left(n\sqrt{\log n}\right)$$

**Sparse IFFT**

$$O\left(n\sqrt{\log n}\right)$$

# QuickSync Synchronization



# Formal Analysis

**Theorem:** (informally restated)

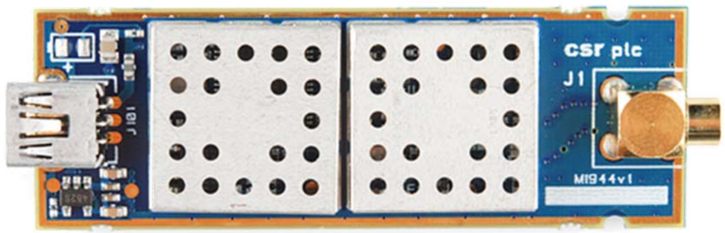
*For any SNR QuickSync achieves the same accuracy as FFT-Based synchronization and has a complexity of* $O(n\sqrt{\log n})$ *where* $n$ *is the number of samples in the code*

*For moderately low SNR (i.e. noise is bounded by* $O(n/\log^2 n)$ *), QuickSync has* $O(n)$ *complexity*

# Rest of this Talk

- GPS Primer
- Our GPS Synchronization Algorithm
- Empirical Results

# Setup




SciGe GN3S Sampler


USRP Software radios

- Traces are collected both US and Europe
- Different locations: urban – suburban
- Different weather conditions: cloudy – clear

# Compared Schemes

- QuickSync Synchronization
- FFT-Based Synchronization

# Metrics

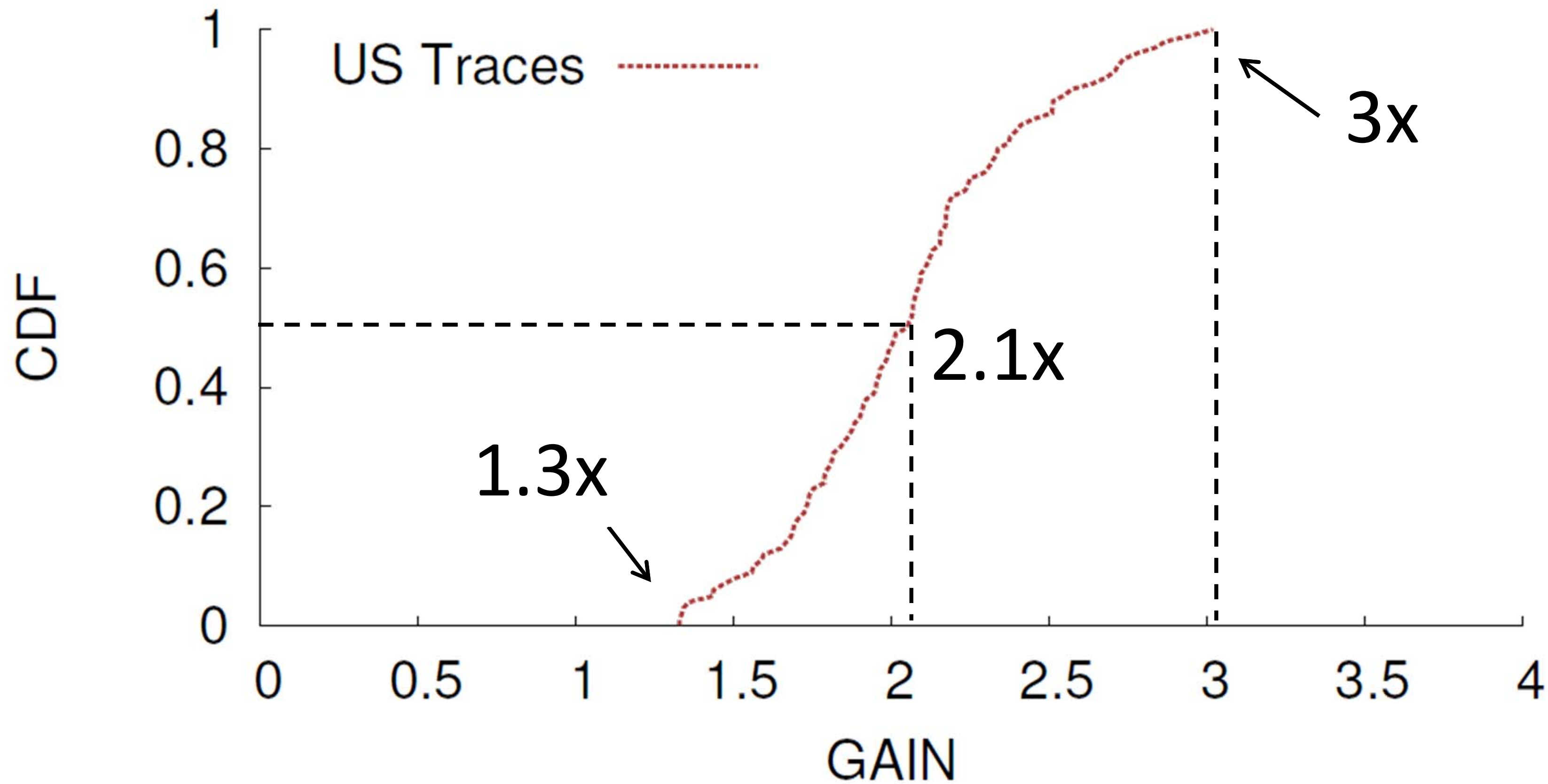
- Hardware implementations

$$\text{Multiplication Gain} = \frac{\text{Multiplications of baseline}}{\text{Multiplications of QuickSync}}$$

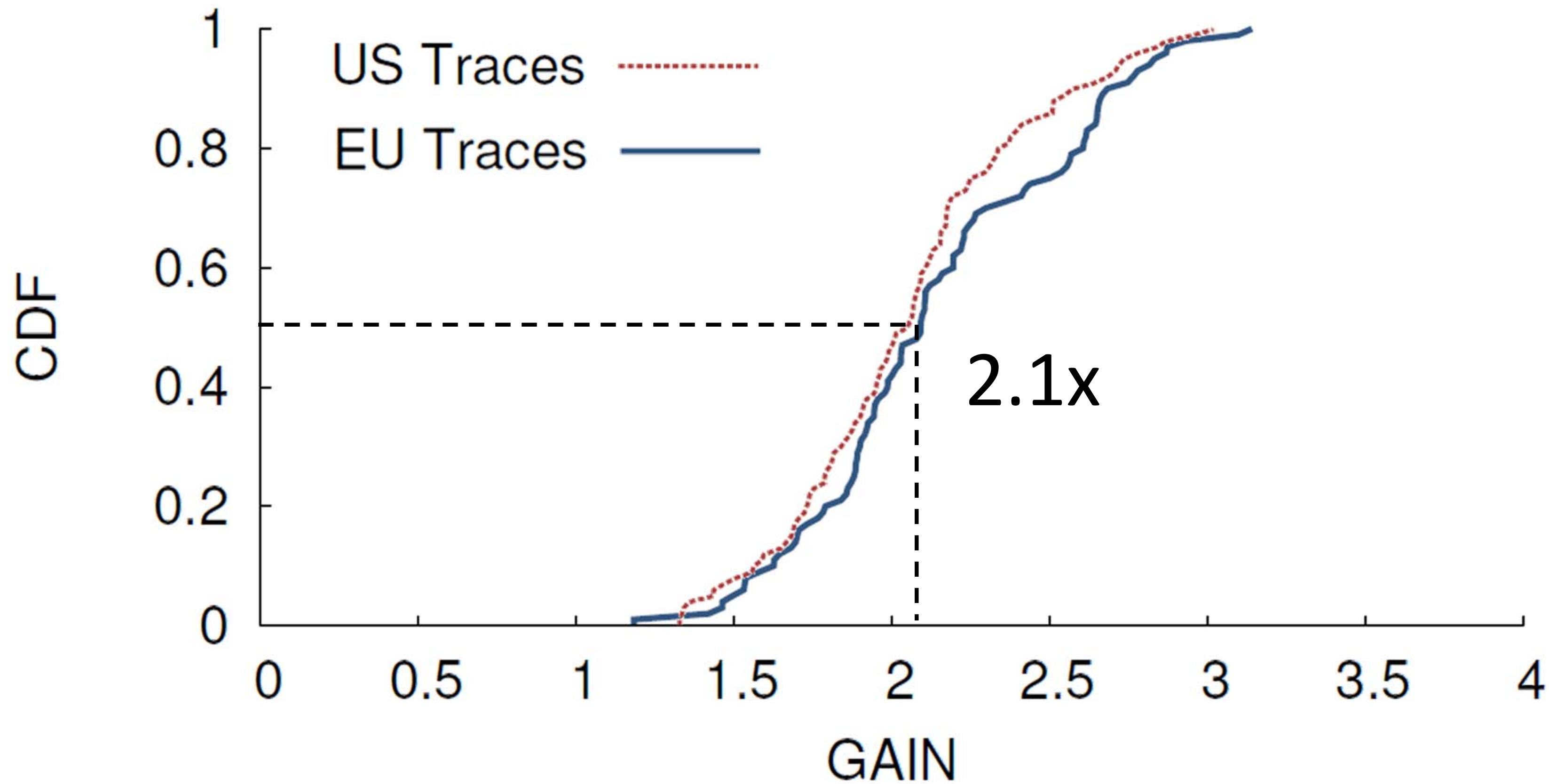
- Software implementations

$$\text{FLOPS Gain} = \frac{\text{FLOPS of baseline}}{\text{FLOPS of QuickSync}}$$

# Multiplication Gain

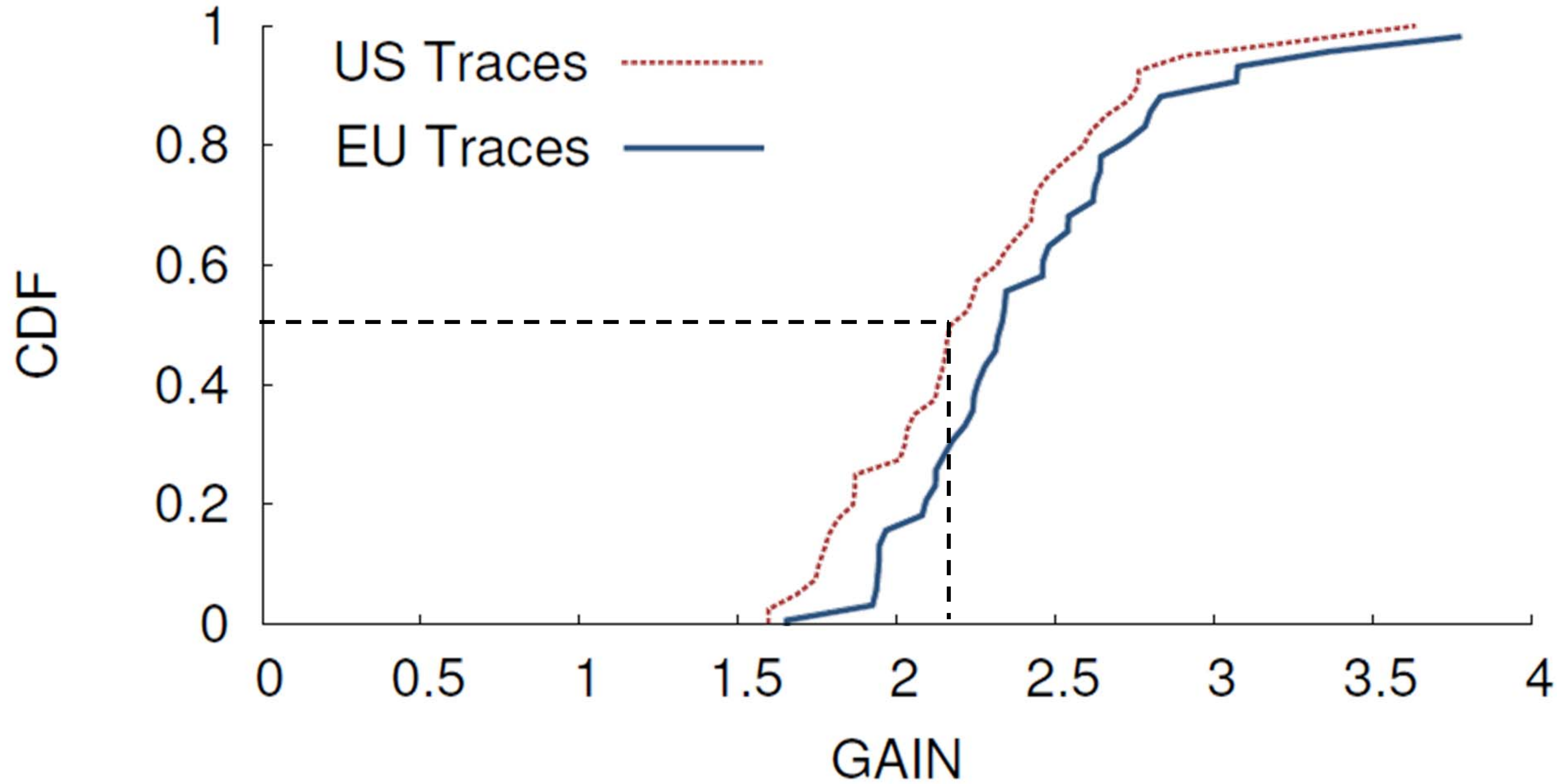


# Multiplication Gain



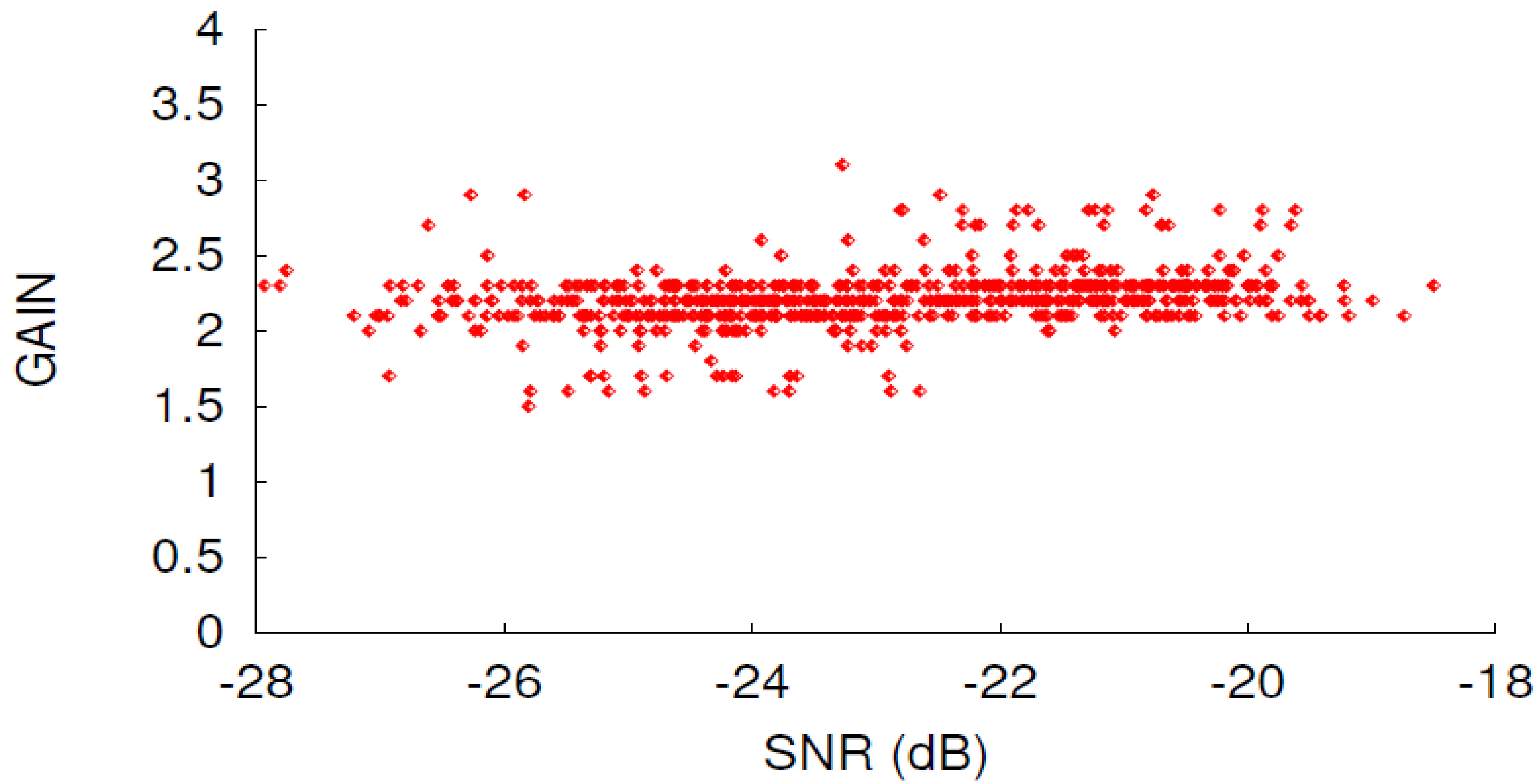

**QuickSync provides an average gain of 2.1x**

# FLOPS Gain




**QuickSync provides an average gain of 2.2 ×**

# Does the Gain Depend on the GPS SNR?




**QuickSync improves over FFT-Based for the whole range of GPS SNRs**

# Related Work

- Past work on GPS [NC91, SA08, RZL11]
  - QuickSync presents the fastest algorithm to date

- Sparse FFT Algorithms [GMS05, HKIP12a, HKIP12b]
  - QuickSync's bucketization leverages duality
    → reduces the complexity of both stages in GPS

# Conclusion

- Fastest GPS synchronization algorithm
  - $O\left(n\sqrt{\log n}\,\right)$ for any SNR
  - $O(n)$ for moderately low SNR
- Empirical results show an average 2x gain
- QuickSync applies to general synchronization tasks beyond GPS